

# INDUCTION DAY 7: UNIT REVIEW & MAXWELL

## BIG IDEAS TO REMEMBER

- $\Phi = BA \cos\theta$  (Wb); closed surfaces always net zero (no monopoles).
- $\mathcal{E} = -\mathbf{N} \cdot d\Phi/dt$ ; Lenz's minus = energy conservation. Change B, A, or  $\theta$  — one law covers all three.
- **Motional:**  $\mathcal{E} = BLv$ ; drag  $F = B^2 L^2 v/R$ ;  $P = Fv = \mathcal{E}^2/R$  exactly.
- **Curly E:**  $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi/dt$  — no potential map;  $(r/2)dB/dt$  inside a ramping solenoid.
- **Inductors:**  $\mathcal{E}_L = -L \cdot dI/dt$ ;  $\tau = L/R$ ;  $U = \frac{1}{2}LI^2$ ; open-switch  $\rightarrow$  bare-wire over  $\sim 5\tau$ .
- **Generator:**  $\mathcal{E} = NBA\omega \sin(\omega t)$  — the flux cosine, differentiated by rotation.

## A Direction & Concept Rapid-Fire

One line each: **(a)** magnet's N pole pulled AWAY from a loop (viewed from the magnet's side) — induced current direction? **(b)** at  $t = 0$ , a charging inductor acts like what element? A charging capacitor? **(c)** why does  $\tau = L/R$  SHRINK when R grows, while  $\tau = RC$  grows? **(d)** where does a generator's electrical energy come from?

## MIXED PRACTICE

### B Faraday Chain

A 300-turn coil ( $A = 0.02 \text{ m}^2$ ,  $R = 6.0 \, \Omega$ ) sits in a field ramping at  $0.5 \text{ T/s}$ . Find  $\epsilon$ , then  $I$ , then the power dissipated.

### C Rails Chain

$B = 0.5 \text{ T}$ ,  $L = 0.8 \text{ m}$ ,  $v = 4.0 \text{ m/s}$ ,  $R = 0.4 \, \Omega$ . Find  $\epsilon$ ,  $I$ , the drag force, and verify  $Fv = \epsilon^2/R$ .

### D LR Chain

$L = 2.0 \text{ H}$  carrying  $3.0 \text{ A}$  ( $U = \frac{1}{2}LI^2$  — compute it) discharges through  $R = 8.0 \, \Omega$ . Find  $\tau$ , the current at  $t = \tau$ , and where the stored joules end up.

## THE COURSE IN FOUR EQUATIONS

### MAXWELL'S EQUATIONS — YOU NOW OWN ALL FOUR

- **Gauss (E):**  $\oint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enc}}/\epsilon_0$  — charges source E (Unit 8).
- **Gauss (B):**  $\oint \mathbf{B} \cdot d\mathbf{A} = 0$  — no monopoles; B-lines close (Unit 13, Day 1).
- **Faraday:**  $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt$  — changing B curls E (Unit 13).
- **Ampère(–Maxwell):**  $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} (+ \mu_0 \epsilon_0 \cdot d\Phi_E/dt)$  — currents (and changing E) curl B (Unit 12).

Chase Faraday and Ampère–Maxwell into empty space and they feed each other: a wave of E and B, traveling at  $1/\sqrt{\mu_0 \epsilon_0} = 3 \times 10^8$  m/s. Light. That's the whole course's punchline.

### E Short FRQ: Derive the LR Rise

A battery  $\mathcal{E}$ , resistor R, and inductor L are switched on at  $t = 0$ .

- a. **Write** the loop rule:  $\mathcal{E} - IR - L \cdot di/dt = 0$ .

- b. **Verify** by substitution that  $I(t) = (\mathcal{E}/R)(1 - e^{-(Rt/L)})$  satisfies it, with  $I(0) = 0$ .

- c. **Explain** physically why I starts at zero and ends at  $\mathcal{E}/R$ .

### F Reflection

Which of these can you least reliably do on your own — Lenz directions, motional-EMF chains, induced electric fields, or LR exponentials? Name it, and write your plan to fix it before the test:

**Test Reminder:** induction responds to CHANGE, never amount; Lenz before algebra on every direction question;  $\tau = L/R$  (R below!); and the generator's  $\omega$  appears twice — amplitude AND frequency.