

INDUCTION DAY 4: INDUCED ELECTRIC FIELDS

WARMUP Who Pushes the Charges?

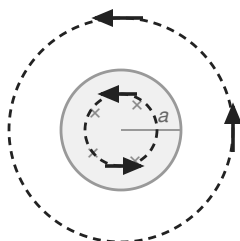
In yesterday's moving bar, $q\mathbf{v} \times \mathbf{B}$ pushed the charges. But a STATIONARY loop around a ramping solenoid also drives current — nothing moves! What must exist at the loop to push those charges?

FARADAY'S LAW, DEEPEST FORM

$$\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt$$

Changing B creates a CURLY electric field — closed field lines, no charges needed. Inside a ramping solenoid (radius a): symmetry gives $E \cdot 2\pi r = (dB/dt)\pi r^2$, so $\mathbf{E} = (r/2)d\mathbf{B}/dt$ inside; $\mathbf{E} = (a^2/2r)d\mathbf{B}/dt$ outside (nonzero even where $B = 0$!).

No potential map: the round-trip integral is nonzero, so this E has no V . Induced EMFs are energy sources, not potential differences.



shaded disk: the solenoid's cross-section —
 B points into the page ($\times$) and is GROWING

dashed: circular paths at $r < a$ and $r > a$ —
the arrows show the induced E on each

Top view of the ramping solenoid. The induced E forms closed circles — it exists at $r > a$ even though B there is zero.

WE DO Exercise 1: Inside the Solenoid

A solenoid of radius $a = 0.10$ m ramps at $dB/dt = 2.0$ T/s. **(a)** E at $r = 0.05$ m: $(r/2)(dB/dt) =$ _____. **(b)** The EMF around that $r = 0.05$ m circle: $E \cdot 2\pi r =$ _____.

WE DO Exercise 2: Outside the Solenoid

Same solenoid ($a = 0.10$ m, $dB/dt = 2.0$ T/s). **(a)** A loop at $r = 0.20$ m encloses flux only through the solenoid's cross-section πa^2 . $E = (a^2/2r)(dB/dt) = \underline{\hspace{2cm}}$. **(b)** B at that radius is essentially ZERO — yet E is not. One sentence on how.

YOU DO Exercise 3: A Charge Rides the Loop

A $+2.0 \mu\text{C}$ charge is carried once around the $r = 0.05$ m circle from Exercise 1. Find the work done on it by the induced field: $W = q\mathcal{E}$. (For a Coulomb field this round trip would cost ZERO — say why.)

CALC Exercise 4: Derive Both Profiles

Run the census twice: for $r < a$ (enclosed flux $\pi r^2 B$) and $r > a$ (enclosed flux $\pi a^2 B$). Derive $E(r)$ in each region, check they agree at $r = a$, and sketch E vs r — the same rise-then-fall profile as B inside a thick wire.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Inside

Solenoid $a = 0.2$ m ramping at 1.5 T/s: find E at $r = 0.1$ m.

P2 Outside

The same solenoid: find E at $r = 0.4$ m.

P3 EMF on a Ring

A wire ring of radius 0.1 m sits inside that solenoid, centered. Find the EMF driving it: $\mathcal{E} = \pi r^2 \cdot dB/dt$.

P4 Two Parents of E

Compare the two electric fields of this course: the Coulomb field of charges and the induced field of changing flux. For each: field-line shape, $\oint \mathbf{E} \cdot d\mathbf{l}$, and whether a potential V exists. One table, three rows.