CIRCUITS DAY 5: RC CIRCUITS

WARMUP The Feedback Loop

A charged capacitor discharges through a resistor. The charge drives the current, and the current drains the charge. Predict the shape of q(t): **steady fall to zero / fast at first, then ever slower / slow at first, then speeding up**? Circle one, and use that feedback to say why.

RC CIRCUITS: THE EXPONENTIAL UNIT

Discharge: loop rule $q/C = IR$ with $I = -dq/dt$ gives $dq/dt = -q/RC$, so

$$q(t) = Q_0 e^{(-t/\tau)}, \quad \tau = RC \text{ (seconds — check: } \Omega\cdot F = s)$$

Charging toward a battery ε: $q(t) = C\varepsilon(1 - e^{(-t/\tau)})$ — same τ, approaching full instead of empty.

Landmarks: at $t = \tau$, 63% of the change is done; at 5τ, 99.3%. I, V, and q all share the same exponential clock.

WE DO Exercise 1: The Clock

C = 2.0 μF charged to 12 V discharges through R = 1.0 MΩ. **(a)** $\tau = RC$ = ________ s. **(b)** $I_0 = V_0/R$ = ________ μA. **(c)** Total charge released = $CV_0 = I_0\tau$ = ________ μC (both products!).

READING THE EXPONENTIAL

WE DO **Exercise 2: Evaluate the Curve**

For Exercise 1's discharge (Q_0 = 24 μC, τ = 2.0 s): **(a)** q at t = 2.0 s: _________ μC. **(b)** q at t = 4.0 s: _________ μC. **(c)** When is q = 12 μC? t = τ ln 2 = _________ s.

YOU DO **Exercise 3: Charging Up**

The same RC pair (τ = 2.0 s) now CHARGES from a 12 V battery: q(t) = 24(1 – e^(–t/2)) μC. Find q at t = 2.0 s, and the current at t = 0 (the capacitor starts as a bare wire!).

CALC **Exercise 4: Derive the Discharge**

Start from the loop rule q/C – IR = 0 with I = –dq/dt. Separate variables (dq/q = –dt/RC), integrate both sides, and exponentiate to reach q(t) = Q_0e^(–t/RC). Show where the initial condition enters.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 The Time Constant

$R = 500\ k\Omega$, $C = 4.0\ \mu F$. Find τ.

P2 Initial Current

A 3.0 μF capacitor at 9.0 V discharges through 1.5 MΩ. Find I_0 in μA.

P3 One Tau Later

For P2's circuit, how much charge remains on the capacitor at $t = \tau$? ($q = Q_0 e^{-1}$.)

P4 Where the Energy Went

P2's capacitor stored $U = \frac{1}{2}CV^2$ initially. Compute U — and state where ALL of it is by $t \gg \tau$. (Day 2's integral $\int I^2R\,dt$ collects exactly this much heat; no derivation needed, just the argument.)