

CIRCUITS DAY 4: KIRCHHOFF & REAL BATTERIES

WARMUP Unit 9 Pays Off Again

On the potential map, any closed walk nets zero: $\oint \mathbf{E} \cdot d\mathbf{l} = 0$. A circuit loop IS a closed walk on that map. What must the voltage changes around any loop therefore sum to?

KIRCHHOFF'S TWO RULES + THE REAL BATTERY

Junction rule (charge conservation): current in = current out.

Loop rule (single-valued potential): $\Sigma \Delta V$ around any loop = 0. Battery – to +: $+\epsilon$. Through a resistor WITH the current: $-IR$.

Real battery: an EMF ϵ in series with internal resistance r . Terminal voltage **$V_{\text{term}} = \epsilon - Ir$** sags under load; $I = \epsilon / (R + r)$.

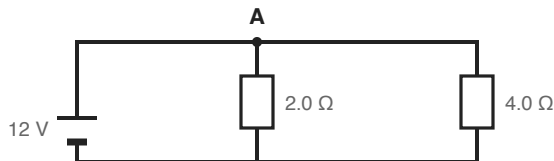
WE DO Exercise 1: One Loop, Fully Audited

$\epsilon = 12 \text{ V}$, $r = 0.5 \Omega$, external $R = 5.5 \Omega$. **(a)** $I = \epsilon / (R + r) =$ _____. **(b)** $V_{\text{term}} = \epsilon - Ir =$ _____. **(c)** Write the full loop sum $+12 - I(0.5) - I(5.5)$ and show it hits zero.

TWO LOOPS, AND BATTERY ECONOMICS

WE DO Exercise 2: Two-Loop Setup

A 12 V battery feeds node A; from A, a $2.0\ \Omega$ resistor returns to the battery, and separately a $4.0\ \Omega$ resistor also returns. Write the junction equation at A and one loop equation per branch, then solve: $I_2 = \underline{\hspace{2cm}}$, $I_4 = \underline{\hspace{2cm}}$, $I_{\text{battery}} = \underline{\hspace{2cm}}$.



YOU DO Exercise 3: The Sagging Battery

A battery reads 9.0 V open-circuit but only 8.0 V while delivering 2.0 A. Find its internal resistance r .

CALC Exercise 4: Maximum Power Transfer

P delivered to R is $P(R) = \varepsilon^2 R / (R+r)^2$. Differentiate with respect to R , set $dP/dR = 0$, and show the load that extracts maximum power is $R = r$. (Quotient rule; the algebra collapses fast.)

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Loop Current

$\varepsilon = 9.0 \text{ V}$, $r = 0.5 \text{ } \Omega$, $R = 4.0 \text{ } \Omega$. Find I .

P2 Terminal Voltage

For P1's circuit, find the terminal voltage of the battery.

P3 Waste Heat

For P1's circuit, find the power wasted inside the battery (I^2r) and the power delivered to R . What fraction of the total reaches the load?

P4 Find the EMF

A battery with $r = 0.25 \text{ } \Omega$ delivers 4.0 A to a $2.75 \text{ } \Omega$ load. Work backward around the loop to find ε — then state what a voltmeter across the terminals reads during this delivery.