CHARGES DAY 6: GAUSS'S LAW APPLICATIONS

WARMUP The Recipe, Recited

Write the three steps of the Gauss recipe from memory (surface choice → geometric flux → solve). What property must the charge distribution have for step 1 to work at all?

SPHERICAL SHELL: TWO REGIMES

A uniform shell of total charge Q, radius R. Gaussian sphere of radius r:

Outside ($r > R$): $q_{enc} = Q$ → $E \cdot 4\pi r^2 = Q/\varepsilon_0$ → $\mathbf{E = kQ/r^2}$ — the whole shell acts like a point at its center (the shell theorem, in two lines — a result Newton needed a hard integral to prove).

Inside ($r < R$): $q_{enc} = 0$ → **$E = 0$** — everywhere inside, exactly.

WE DO Exercise 1: Shell Numbers

A shell of radius 2.0 m carries Q = +8.0 μC. Find E at: **(a)** r = 4.0 m; **(b)** r = 2.5 m; **(c)** r = 1.0 m. One of these needs no arithmetic at all.

SOLID SPHERE: THE INSIDE GROWS LINEARLY

Uniformly charged SOLID sphere (charge Q, radius R). Inside, the Gaussian sphere encloses only the fraction $q_{enc} = Q\cdot(r^3/R^3)$:

$E\cdot 4\pi r^2 = Q\, r^3/(\varepsilon_0 R^3)$ → $\mathbf{E = kQr/R^3}$ (inside) — zero at the center, rising linearly to kQ/R^2 at the surface, then falling as kQ/r^2 outside.

CYLINDERS FOR LINES, PILLBOXES FOR PLANES

INFINITE LINE OF CHARGE

Wrap a line (λ, C/m) in a Gaussian CYLINDER of radius r and length ℓ. Field pierces only the curved wall (area $2\pi r\ell$); the end caps catch nothing:

$$E \cdot 2\pi r\ell = \lambda\ell/\varepsilon_0 \rightarrow \mathbf{E = \lambda/(2\pi\varepsilon_0 r) = 2k\lambda/r}$$

Day 3's hardest integral, reproduced in two lines. The ℓ cancels — as it must for an infinite line.

INFINITE SHEET OF CHARGE

Poke a small "pillbox" cylinder THROUGH a sheet (σ, C/m²), caps of area A on both sides. Field exits both caps; the wall catches nothing:

$E \cdot (2A) = \sigma A/\varepsilon_0 \rightarrow$ **$E = \sigma/(2\varepsilon_0) = 2\pi k\sigma$** — constant, distance-free: Day 4's disk limit.

WE DO **Exercise 2: Line Numbers**

A long straight wire carries $\lambda = 3.0$ μC/m. **(a)** E at r = 0.5 m. **(b)** E at r = 1.5 m. **(c)** Note the falloff: tripling r did what to E — and why is that gentler than a point charge's falloff?

YOU DO **Exercise 3: Sheet Numbers**

A large sheet carries $\sigma = 4.0$ μC/m². Find E near it — and state E twice as far away.

CONDUCTORS: E = 0 INSIDE

E = 0 INSIDE A CONDUCTOR (ELECTROSTATICS)

In a conductor, charge moves freely. If any field existed inside, charges would flow — and they DO flow, piling up on the surfaces until their own field exactly cancels the intruder. At equilibrium: **E = 0 everywhere inside the metal**. Consequences via Gauss:

- Any net charge on a conductor lives entirely on its **surface** (a Gaussian surface just under the skin encloses zero).
- Just OUTSIDE the surface: $E = \sigma/\varepsilon_0$, perpendicular to the metal (twice a lone sheet's $\sigma/2\varepsilon_0$ — the Gaussian pillbox has flux through only its outer cap, because E = 0 in the metal).
- A cavity inside a neutral conductor is **shielded**: external static fields never reach it.

WE DO Exercise 4: The Charged Conducting Sphere

A solid conducting sphere of radius 1.0 m carries +6.0 µC. Find E at: **(a)** r = 0.5 m (inside the metal); **(b)** the surface, r = 1.0 m; **(c)** r = 3.0 m. Sketch E(r) from 0 to 4 m.

CALC Exercise 5: Point Charge Inside a Neutral Shell

A point charge +q sits at the center of a NEUTRAL conducting shell (inner radius a, outer b). Use Gaussian spheres in the metal (where E must be 0) to find the induced charge on: **(a)** the inner surface; **(b)** the outer surface. **(c)** What is E outside the shell (r > b)?

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Shell, Three Radii

A shell ($R = 1.5$ m, $Q = +6.0$ μC): find E at $r = 1.0$ m, at $r = 1.5$ m (just outside), and at $r = 3.0$ m.

P2 Inside the Solid Sphere

A solid insulating sphere ($R = 2.0$ m, $Q = +8.0$ μC, uniform): find E at $r = 1.0$ m using $E = kQr/R^3$, and at $r = 4.0$ m.

P3 Line vs Point

A wire carries $\lambda = 2.0$ μC/m. Find E at 0.5 m and at 2.0 m. Then, for a POINT charge, what would quadrupling r do to E — versus what it did here?

P4 Inner and Outer Surface Charges

A −3.0 μC point charge sits at the center of a neutral conducting shell. State the charge on the inner surface, the charge on the outer surface, and E at a point inside the metal — each with its one-line Gaussian justification.