CHARGES DAY 5: FLUX & GAUSS'S LAW

WARMUP Counting Rain (3 min)

Rain falls straight down. You hold a hoop of area A. How does the amount of rain THROUGH the hoop change if you: **(a)** double the hoop's area? **(b)** tilt the hoop 60° from face-on? **(c)** hold it edge-on to the rain?

ELECTRIC FLUX: FIELD THROUGH A SURFACE

Swap rain for field lines and you have **flux**:

$$\Phi = E \cdot A \cdot \cos\theta \quad \rightarrow \text{in general} \quad \Phi = \int E \cdot dA$$

θ is measured from the surface's NORMAL. Units: (N/C)(m^2) = **N·m^2/C**. The integral form is another accumulation: chop the surface into dA patches, count E·cosθ·dA through each, add. For a CLOSED surface, outward is positive — flux out counts +, flux in counts –.

WE DO Exercise 1: Flat-Surface Flux

A uniform field E = 500 N/C points +x. Find the flux through a 2.0 m^2 flat surface whose normal points: **(a)** along +x; **(b)** 60° from +x; **(c)** along +y. Attach units to each.

GAUSS'S LAW: THE FLUX DOESN'T CARE ABOUT R

THE SPHERE EXPERIMENT

Wrap a point charge q in an imaginary sphere of radius r. On that sphere $E = kq/r^2$ everywhere, pointing straight out ($\cos\theta = 1$ on every patch), so:

$$\Phi = E\cdot(4\pi r^2) = (kq/r^2)(4\pi r^2) = \mathbf{4\pi kq} \text{ — the } r^2 \text{ cancels!}$$

Grow the sphere and the field weakens exactly as fast as the area grows. The flux depends only on the charge inside. That is **Gauss's law**:

$$\boldsymbol{\Phi_{closed} = q_{enc}/\varepsilon_0} \quad (\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2\text{; note } 4\pi k = 1/\varepsilon_0)$$

It holds for ANY closed surface, any shape — charges outside contribute zero net flux (in one side, out the other).

WE DO **Exercise 2: Verify the Cancellation**

A +4.0 μC charge sits at the center of a sphere. **(a)** For r = 1.0 m: E = ________ N/C, A = $4\pi r^2$ = ________ m², Φ = ________ N·m²/C. **(b)** Repeat for r = 2.0 m. **(c)** The two fluxes match. Verify both equal $4\pi kq$.

YOU DO **Exercise 3: Read the Enclosure**

A closed surface encloses charges +5.0 μC and –2.0 μC; a +7.0 μC charge sits OUTSIDE. **(a)** q_{enc} = ________. **(b)** $\Phi = q_{enc}/\varepsilon_0$ = ________. **(c)** Why does the outside charge contribute nothing net, even though its field pierces the surface?

TURNING GAUSS AROUND: FINDING E FROM SYMMETRY

THE THREE-STEP RECIPE

- **(1) Choose a Gaussian surface** matching the symmetry — sphere for points/spheres, cylinder for lines/planes — so E is constant on it and parallel to dA.
- **(2) Write the flux geometrically:** Φ = E × (area where field pierces).
- **(3) Set it equal to q_enc/ε_0** and solve for E.

Day 3's hard-won infinite line 2kλ/y falls out in two lines tomorrow. Gauss's shortcut demands symmetry: no symmetry, no shortcut.

WE DO **Exercise 4: Sphere Around a Point, Formally**

Run the recipe for a point charge q: choose the sphere of radius r, write $\Phi = E\cdot 4\pi r^2$, set equal to q/ε_0, and solve for E. What familiar law did Gauss just hand back?

CALC **Exercise 5: Flux as an Accumulation**

A field E = 200x N/C (growing with x) points +x through a cube of side 1.0 m with faces at x = 0 and x = 1.0 m. **(a)** Flux IN through the x = 0 face? **(b)** Flux OUT through x = 1.0 m? **(c)** Net flux, and the enclosed charge it implies ($q\_enc = \varepsilon_0\Phi$).

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Tilted Loop

$E = 300$ N/C uniform; a 0.50 m^2 loop's normal makes 60° with E. Find the flux.

P2 What's Inside?

The net flux through a closed surface is 5.65×10^5 N·m^2/C. Find the enclosed charge.

P3 Two Spheres, One Charge

A +2.0 μC charge is wrapped by spheres of radius 1.0 m and 3.0 m. Compute the flux through each (show they match), then E on each surface (show they don't).

P4 The Cube Trick

A +6.0 μC charge sits at the CENTER of a cube. **(a)** Total flux through the cube (no integration!). **(b)** Flux through ONE face, by symmetry. **(c)** Why would this be miserable to compute with $\int E\cdot dA$ directly?