CHARGES DAY 4: FIELDS BY INTEGRATION II

WARMUP Day 3 Bridge

Point P sits on the axis of a charged ring, height z above its center. A dq slice sits somewhere on the rim. **(a)** How far is EVERY slice from P? **(b)** Why does that make the ring integral easier than yesterday's rod?

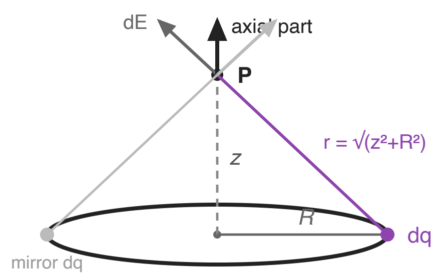
RING ON ITS AXIS

Every dq sits at the same distance $r = \sqrt{z^2 + R^2}$ from P — and by symmetry, the sideways components of dE cancel in pairs around the ring. Only the axial component $\cos\theta = z/r$ survives:

$$\mathbf{E = kQz / (z^2 + R^2)^{3/2}} \quad \text{(along the axis)}$$

Because r is CONSTANT, the "integral" needs no integral table — pull everything out and $\int dq = Q$. The symmetry of the geometry does the work the calculus would otherwise do.

Limit checks: z = 0 (center) → E = 0 by symmetry ✓; z ≫ R → kQ/z^2, a point charge ✓.



Every slice sits the same distance r from P; the sideways parts of dE cancel in mirror pairs, the axial parts add.

WE DO Exercise 1: Ring Numbers

A ring of radius 1.5 m carries $Q = +5.0\ \mu C$. Find E on the axis at z = 2.0 m: **(a)** $r^2 = z^2 + R^2$ = _________; **(b)** $E = kQz/(r^2)^{3/2}$ = _________ N/C. **(c)** Which way does it point, and what flips if Q is negative?

READING THE RING FORMULA

WE DO **Exercise 2: The Two Ends of the Axis**

For the same ring ($R = 1.5$ m, $Q = 5.0\ \mu C$): **(a)** evaluate E at $z = 0$ and explain the answer physically. **(b)** Evaluate the point-charge approximation kQ/z^2 at $z = 10$ m and the exact formula at $z = 10$ m — how close? **(c)** Between those extremes E must peak somewhere; the calculus says $z = R/\sqrt{2} \approx 1.06$ m. Compute E at $z = 1.0$ m, at 1.06 m, and at 1.5 m. The first two nearly tie — why is that near-equality itself evidence you're AT the peak? — and the drop by $z = 1.5$ m completes the rise-peak-fall.

YOU DO **Exercise 3: Your Ring**

A ring of radius 2.0 m carries $Q = -8.0\ \mu C$. Find the field on the axis at $z = 1.5$ m (magnitude and direction).

THE DISK: RINGS ALL THE WAY OUT

INTEGRATE RINGS TO BUILD A DISK

A charged disk (surface density σ, C/m²) is nested rings: the ring at radius s with width ds carries $dq = \sigma \cdot 2\pi s \cdot ds$. Feeding each ring into the ring formula and integrating from 0 to R gives:

$$E = 2\pi k\sigma\,[1 - z/\sqrt{(z^2 + R^2)}]$$

Limit checks again: **($z \gg R$)** → kQ/z^2 with $Q = \sigma\pi R^2$ (point charge ✓); **($R \to \infty$)** → $E = 2\pi k\sigma = \sigma/(2\varepsilon_0)$, a CONSTANT — the infinite sheet's field doesn't fade with distance at all. That constant is the backbone of capacitors in Unit 10.

WE DO **Exercise 4: Disk Numbers**

A disk of radius 3.0 m carries σ = 2.0 μC/m². Find E on the axis at z = 4.0 m: **(a)** $\sqrt{(z^2 + R^2)}$ = _________; **(b)** bracket $[1 - z/\sqrt{(z^2+R^2)}]$ = _________; **(c)** $E = 2\pi k\sigma \times$ bracket = _________ N/C.

CALC **Exercise 5: The Sheet Limit**

For σ = 2.0 μC/m²: **(a)** compute the infinite-sheet field $E = 2\pi k\sigma$. **(b)** At what fraction of that value is the disk field of Exercise 4 (z = 4.0 m, R = 3.0 m)? **(c)** One sentence: why does an infinite sheet's field NOT weaken with distance? (Think about what enters your $1/r^2$ view as you back away.)

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Ring, Straight Up

A ring ($R = 2.0$ m, $Q = +6.0\ \mu C$) — find E on its axis at $z = 1.5$ m.

P2 Limit Drill

For P1's ring, compute kQ/z^2 at $z = 20$ m and the exact formula at $z = 20$ m. State the percent difference — and why it's small.

P3 Disk vs Sheet

A disk ($R = 4.0$ m, $\sigma = 1.0\ \mu C/m^2$) — find E at $z = 3.0$ m, and the infinite-sheet value $2\pi k\sigma$ for comparison.

P4 Build the Disk Integral (Setup Only)

Starting from the ring result $E_{ring} = kz \cdot dq/(z^2 + s^2)^{3/2}$ for a ring of radius s: write dq for the ring of width ds on a disk of density σ, substitute, and write the full integral for the disk (limits included). Do not evaluate — but state the u-substitution that would crack it.