CHARGES DAY 3: FIELDS BY INTEGRATION I

WARMUP Unit 4 Bridge

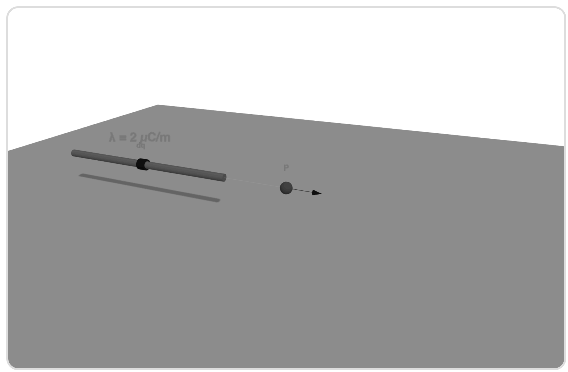
A rod of length 3.0 m has linear CHARGE density $\lambda = 2.0\ \mu C/m$ (uniform). **(a)** What are the units of one $\lambda \cdot dx$ strip? **(b)** Total charge on the rod? (Same machine as mass strips — new payload.)

The dm machine, loaded with charge. In Unit 4 you chopped rods into $dm = \lambda\, dx$ slices to weigh them and find balance points. Today the SAME slices carry charge: $dq = \lambda\, dx$ — and each dq is a tiny point charge whose field you already know: $dE = k \cdot dq/r^2$. Superposition says add them all. Adding infinitely many tiny contributions is an integral. That is the whole day.

THE FIELD INTEGRAL

$$E = \int dE = \int k \cdot dq/r^2$$

Recipe: **(1)** slice the object into dq pieces; **(2)** write each piece's distance r to the field point; **(3)** check by symmetry which components survive; **(4)** integrate over the object.



A dq slice mid-rod and the field point P — sweep the slice along the rod and each contribution adds to E.

WE DO Exercise 1: Set It Up Together

A rod of charge lies on the x-axis from x = –3.0 m to 0, with uniform $\lambda = +2.0\ \mu C/m$. Point P sits at x = +1.0 m. For a slice dq at position x: **(a)** dq = _________ dx. **(b)** Its distance to P: r = _________. **(c)** Which way does dE from EVERY slice point at P — and why does that make this integral scalar-friendly?

RUN THE INTEGRAL

THE ROD-ON-AXIS RESULT

With the rod on [–L, 0] and P at x = a:

$$E = \int_{-L}^{0} k\lambda\, dx/(a - x)^2 = k\lambda\, [1/(a - x)]_{-L}^{0} = \mathbf{k\lambda\, (1/a - 1/(a+L))}$$

Two sanity checks that every field integral must pass: **(near)** as $a \to 0$ it blows up like a point charge you're touching; **(far)** for $a \gg L$, $1/a - 1/(a+L) \approx L/a^2$, so $E \approx k(\lambda L)/a^2 = kQ/a^2$ — from far away the rod IS a point charge. If your integral fails a limit check, it is wrong.

WE DO **Exercise 2: Numbers In**

$\lambda = 2.0\ \mu C/m$, L = 3.0 m, a = 1.0 m. **(a)** Evaluate $E = k\lambda(1/a - 1/(a+L))$ step by step. **(b)** Compare with the "lazy" answer kQ/a^2 treating all Q = 6.0 μC as a point at the rod's center (distance 2.5 m). Which is bigger, and WHY (where does the rod keep its closest charge)?

YOU DO **Exercise 3: Your Rod**

$\lambda = 4.0\ \mu C/m$, L = 2.0 m, P at a = 2.0 m from the near end. **(a)** E at P by the formula. **(b)** Run the far-away check: what does kQ/a^2 give with Q = 8.0 μC at the rod's CENTER (a + L/2 = 3.0 m)? Close?

SYMMETRY: THE COMPONENTS THAT DIE

PERPENDICULAR BISECTOR OF A ROD

Put P a height y above the CENTER of a rod. Each slice at +x has a mirror twin at –x: their horizontal dE components cancel, their vertical components double. You integrate only the surviving component:

$$E = \int k\, dq \cdot \cos\theta / r^2 \quad \text{with} \quad \cos\theta = y/r,\ r = \sqrt{x^2 + y^2}$$

Declare the cancellation BEFORE integrating — it halves the work and it is where exam points live. (The full result, for reference: $E = k\lambda L / (y\sqrt{y^2 + L^2/4})$.)

WE DO **Exercise 4: Argue the Cancellation**

For the bisector setup: **(a)** draw the rod, P above its center, and the dE arrows from a slice pair at ±x; **(b)** mark which components cancel; **(c)** write (don't evaluate) the surviving integral with cosθ substituted.

CALC **Exercise 5: The Infinite-Rod Limit**

In the reference result $E = k\lambda L/(y\sqrt{y^2 + L^2/4})$, let the rod become very long ($L \gg y$). **(a)** Show $E \to 2k\lambda/y$. **(b)** The infinite line's field falls off as $1/y$, not $1/y^2$ — one sentence on why "more rod" compensates one power of distance. (Gauss's law will hand us this same result in two lines on Day 5.)

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Strip Bookkeeping

A rod on [0, 2.0 m] has $\lambda(x) = 3.0x\ \mu C/m^2$. **(a)** Total charge by integration. **(b)** Where is dq largest?

P2 Rod on Axis

$\lambda = 3.0\ \mu C/m$, L = 2.0 m, P at a = 1.0 m from the near end. Find E at P.

P3 Limit Check Drill

For P2's rod: evaluate the far-field approximation kQ/d^2 with all charge at the rod's center (d = 2.0 m), and state whether it over- or under-estimates the true field — with the physical reason.

P4 Set Up the Hard One (No Evaluation)

A rod on [0, L] has the NON-uniform density $\lambda(x) = cx$. P sits on the axis at x = L + a. Write the complete integral for E at P — dq, r, limits — and state which single extra step (compared with today's uniform rod) the integral table would need. Do not evaluate.