CAPACITORS DAY 3: CAPACITORS BY CALCULUS

WARMUP The Recipe Assembles Itself

You already own every ingredient: Gauss's law gives E between the conductors, and $\Delta V = \int E \cdot dr$ turns that field into a potential difference. Write both from memory: E of a point/sphere, and the shell-to-shell result $\Delta V = kQ(1/a - 1/b)$.

CAPACITANCE OF ANY GEOMETRY: THE RECIPE

1) Put ±Q on the conductors. 2) Gauss → E between them. 3) $\Delta V = \int E \cdot dr$ from − to +. 4) **$C = Q/\Delta V$** — Q always cancels.

Spherical (radii $a < b$): $\Delta V = kQ(1/a - 1/b)$ → **$C = ab/[k(b-a)]$**. Send $b \to \infty$: an **isolated sphere** has $C = a/k = 4\pi\varepsilon_0 a$.

Cylindrical (length L): $E = 2k\lambda/r$ → $\Delta V = 2k\lambda \ln(b/a)$ → **$C = L/[2k \ln(b/a)]$**.

WE DO Exercise 1: Spherical, Start to Finish

Inner shell $a = 0.5$ m with $+4.0$ μC, outer shell $b = 2.0$ m with -4.0 μC. **(a)** $\Delta V = kQ(1/a - 1/b)$ = ________ V. **(b)** $C = Q/\Delta V$ = ________ pF.

LIMITS THAT TEACH

WE DO **Exercise 2: The Lonely Sphere**

(a) Isolated sphere, $a = 0.9$ m: $C = a/k =$ _________ pF. **(b)** The entire EARTH ($a = 6.4\times10^6$ m): $C =$ _________ μF. React to how small that is.

YOU DO **Exercise 3: The Thin-Gap Limit**

Spherical capacitor with $a = 1.0$ m, $b = 1.05$ m. Compute $C = ab/[k(b-a)]$ — then explain why a nearly-touching spherical capacitor behaves like a parallel-plate one (what do $4\pi a^2$ and $b-a$ play the role of?).

CALC **Exercise 4: Cylindrical Derivation**

A line/cylinder with $\lambda = 2.0$ μC/m: $E = 2k\lambda/r$. Integrate $\Delta V = \int_a^b E\,dr$ for $b = 2a$ and evaluate: $\Delta V = 2k\lambda \ln 2 =$ _________ V. (Note the logarithm — no power law here.)

PAPER PRACTICE

PRACTICE PROBLEMS

P1 **Shell Walk**

$Q = 6.0\ \mu C$, $a = 0.6$ m, $b = 3.0$ m: find ΔV between the shells.

P2 **Its Capacitance**

For P1's capacitor, find C two ways: $Q/\Delta V$, and $ab/[k(b-a)]$.

P3 **Size a Sphere**

What radius must an isolated sphere have for $C = 50$ pF?

P4 **Energy in the Shells**

Using P1's numbers ($Q = 6.0\ \mu C$, $\Delta V = 72{,}000$ V), find the stored energy $U = \frac{1}{2}Q\Delta V$ in joules.