CAPACITORS DAY 2: ENERGY STORAGE

WARMUP **The First Parcel Rides Free**

Pushing charge onto a capacitor gets harder as charge accumulates. Why does the FIRST parcel of charge take almost no work to deliver, while the LAST parcel takes the most?

THE CHARGING INTEGRAL

Each parcel dq is pushed through the voltage already built: $dW = V(q)dq = (q/C)dq$. Integrate 0 → Q:

$$U = Q^2/2C = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2$$ — one stored energy, three equivalent forms.

The ½ is the triangle under the V = q/C ramp — the average voltage during charging is only V/2.

Where it lives: in the field itself, with density $u = \tfrac{1}{2}\varepsilon_0 E^2$ (J/m³) — multiply by the gap volume Ad to recover U.

WE DO **Exercise 1: One Energy, Two Routes**

A 2.0 μF capacitor is charged to 12 V. **(a)** Q = _____ μC. **(b)** $U = \tfrac{1}{2}CV^2$ = _____ μJ. **(c)** Check with $Q^2/2C$: _____ μJ.

WE DO Exercise 2: The Defibrillator

A defibrillator capacitor: C = 100 μF charged to 2000 V. Find the stored energy in joules — and compare it to a 1 kg mass dropped from 20 m (U = mgh ≈ 200 J).

YOU DO Exercise 3: Pull-Apart Energetics

A charged, DISCONNECTED capacitor stores 60 μJ. The plate gap is then doubled (C halves, Q trapped). **(a)** New U = $Q^2/2C$ = _____ μJ. **(b)** The energy increased — what did the work?

CALC Exercise 4: Derive $U = Q^2/2C$

Write $W = \int_0^Q (q/C)dq$, evaluate it, and sketch the V-vs-q ramp with the work as the area under it. Mark where the "½" lives in your sketch.

PAPER PRACTICE

PRACTICE PROBLEMS

P1 Half C V Squared

C = 3.0 μF at 10 V. Find U in μJ.

P2 From the Charge

Q = 24 μC on a 4.0 μF capacitor. Find U in μJ.

P3 Solve for Volts

A 2.0 μF capacitor must store 100 μJ. What voltage does it need?

P4 The Field Holds It

Plates: A = 0.02 m², d = 1.0 mm, charged to 100 V (C = 177 pF). **(a)** U = ½CV² = _____ μJ. **(b)** E = V/d, then u = $\frac{1}{2}\varepsilon_0 E^2$, then U = u·(Ad). Show both routes land on the same number.