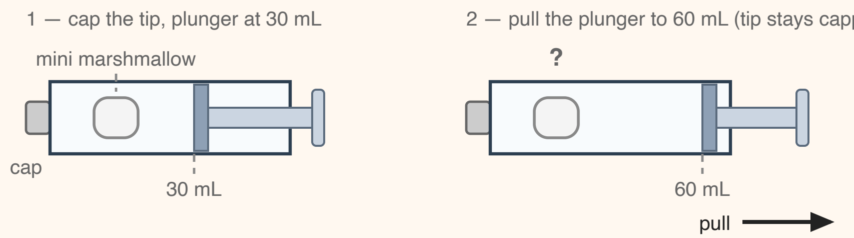
THERMO DAY 3: THE IDEAL GAS LAW

WARMUP The Marshmallow in the Syringe — predict first

Your teacher drops a mini marshmallow into a large syringe, caps the tip so no air can enter or leave, and then **pulls the plunger** from the 30 mL mark out to the 60 mL mark — doubling the space the trapped air fills.



The apparatus. Commit to a prediction before the teacher pulls the plunger.

Prediction — circle one. As the plunger moves from 30 mL to 60 mL, the marshmallow:

(A) shows no visible change — a marshmallow is a solid **(B)** shrinks **(C)** swells

In one line, why did you pick your prediction?

____

THREE GAUGES, ONE MISSING LAW

Day 1 established what temperature reports: T tracks the average translational kinetic energy per particle. Day 2 built pressure from wall collisions and gave molecular speeds through v_{rms}. Each idea described one gauge at a time.

Today: the single equation that ties all four state variables of a gas sample — pressure P, volume V, particle count N, and temperature T — together, so that reading three of them fixes the fourth.

QUICK State Variables and Their SI Units

Fill in the SI unit each quantity carries in gas-law calculations. One of the five carries no unit at all — write "none" for it.

Pressure P: __________ Volume V: __________ Temperature T: __________

Amount in moles n: __________ Particle count N: __________

THE IDEAL GAS LAW — TWO FORMS OF ONE EQUATION

COUNTING PARTICLES OR COUNTING MOLES

$$PV = Nk_BT \qquad PV = nRT$$

Particle form: N = number of molecules, $k_B = 1.38 \times 10^{-23}$ J/K (per particle)

Molar form: n = number of moles, $R = 8.31$ J/(mol·K) (per mole)

A mole is chemistry's dozen: one mole = $N_A = 6.02 \times 10^{23}$ particles, so $N = n\,N_A$. The two forms are the same law — substituting $N = nN_A$ into $PV = Nk_BT$ shows that $R = N_Ak_B$. Nothing else changes; only the counting unit does.

Units check before any algebra: P in pascals (N/m²), V in m³, T in kelvin. Then PV has units $(\mathrm{N/m^2})(\mathrm{m^3}) = \mathrm{N{\cdot}m} = \mathrm{J}$, and nRT has units $(\mathrm{mol})\left(\frac{\mathrm{J}}{\mathrm{mol{\cdot}K}}\right)(\mathrm{K}) = \mathrm{J}$. Both sides are energies — the law balances.

WE DO **The Room-Pressure Tank**

A rigid 0.90 m³ storage tank holds 36.0 mol of nitrogen gas at 310 K. Find the pressure of the nitrogen.

Step 1 — solve for P: $P = \dfrac{nRT}{V} = \dfrac{(36.0\ \mathrm{mol})(8.31\ \frac{\mathrm{J}}{\mathrm{mol{\cdot}K}})(\_\_\_\_\_\ \mathrm{K})}{\_\_\_\_\_\ \mathrm{m^3}}$

Step 2 — compute, carrying units: $P =$ ______________ Pa

YOU DO **The Test Chamber**

A sealed 1.40 m³ test chamber holds 66.0 mol of air at 330 K. Find the pressure of the air in the chamber, carrying units through the computation.

QUICK **By the Carton**

(a) A cylinder holds 2.50 mol of helium. Number of atoms: $N =$ ______________

(b) A flask holds 9.03×10^{23} molecules of CO_2. Moles: $n =$ ______________

(c) Verify the constant conversion: $N_A\,k_B = (6.02 \times 10^{23}\ \mathrm{mol^{-1}})(1.38 \times 10^{-23}\ \mathrm{J/K}) =$ _________ J/(mol·K)

RATIO REASONING: TWO STATES OF ONE SEALED SAMPLE

THE SEALED-SAMPLE SHORTCUT

If the sample is sealed, N does not change — so $\frac{PV}{T} = Nk_B$ is the **same number before and after** any process:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Four-step habit: ① confirm the sample is sealed (N fixed) → ② name which variable the setup holds constant (rigid walls fix V; a slow process in a room fixes T) → ③ convert every temperature to kelvin → ④ set up the ratio.

Kelvin only, and here is why it matters for ratios: "doubling" 37°C to 74°C raises the kelvin temperature from 310 K to only 347 K — a factor of 1.12, not 2. Celsius ratios have no physical meaning because the Celsius zero is the freezing point of water, not the zero of molecular kinetic energy.

WE DO **Heat the Tank**

The rigid 0.90 m³ tank from page 2 (36.0 mol of nitrogen at 310 K, $P_1 = 1.03 \times 10^5$ Pa) is heated to 620 K. Find the new pressure P_2.

(a) Which variables are fixed? __________ and __________ **(b)** So the ratio collapses to: $P_2 = P_1 \times$ ——

(c) Compute P_2, carrying units: _______________ Pa

Back to the marshmallow: the capped syringe seals its air (N fixed), and a slow pull in a room-temperature barrel keeps T nearly fixed. Doubling V from 30 mL to 60 mL therefore **halves** the pressure of the surrounding syringe air. The gas pockets *inside* the marshmallow still push out at close to the old pressure, so the unbalanced push points outward and the marshmallow swells.

Circle one: when the teacher releases the plunger and the trapped air returns to 30 mL, the surrounding pressure **rises / falls** back to its starting value, and the marshmallow shrinks back.

YOU DO **The Slow Squeeze**

Sealed gas in a piston chamber starts at $P_1 = 1.20 \times 10^5$ Pa and $V_1 = 1.50$ m³. The piston compresses the gas slowly to $V_2 = 0.75$ m³ while the surrounding water bath holds the temperature constant. Find P_2, carrying units.

QUICK **Spot the Fixed Variable**

For each sealed sample, name the state variable (besides N) that the setup holds fixed:

(a) a rigid steel tank warming in the sun: __________ **(b)** a capped syringe pulled slowly at room temperature: __________ **(c)** a flexible mylar balloon carried up a warm stairwell (its skin keeps the inside pressure matched to the room's): __________

PRACTICE

1 Pressurize the Vessel

A rigid 1.45 m³ vessel holds 55.0 mol of argon at 305 K. Find the pressure of the argon, carrying units through the computation.

2 Read the Gas Thermometer

A sealed 0.75 m³ container holds 58.0 mol of gas at a pressure of 2.00×10^5 Pa. **(a)** Find the temperature of the gas in kelvin (carry units). **(b)** Convert the gas's temperature to °C.

3 Count the Invisible

Air in a sealed 0.85 m³ drum is at 1.10×10^5 Pa and 340 K. **(a)** Use $PV = Nk_BT$ to find the number of molecules N in the drum. **(b)** Convert the drum's molecule count to moles and confirm the answer with $PV = nRT$.

4 Both Change at Once

A sealed cylinder of gas starts at $P_1 = 1.50 \times 10^5$ Pa, $V_1 = 1.20$ m³, and $T_1 = 315$ K. A piston compresses the gas to $V_2 = 0.70$ m³ while a burner raises its temperature to $T_2 = 630$ K. Find P_2, carrying units through the ratio.

5 Fix the Claim

A sealed rigid tank reads 1.05×10^5 Pa at 42°C. A student says: *"If we heat the tank to 84°C, we double the temperature, so the pressure doubles to 2.10×10^5 Pa."* **(a)** Compute the actual final pressure. **(b)** In one sentence, state what temperature the tank would need to reach (in °C) for the pressure to double.