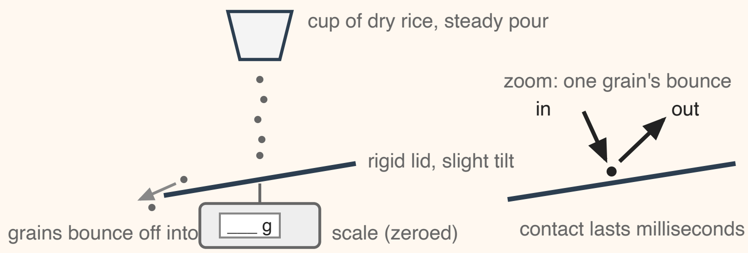
THERMO DAY 2: PRESSURE FROM COLLISIONS & THE RMS SPEED

WHERE WE ARE

Yesterday: temperature tracks the average translational kinetic energy per molecule, $KE_{avg} = \frac{3}{2}k_BT$. Today the other state variable gets its mechanism: **pressure is built one collision at a time**, and v_{rms} puts a number on how fast the colliding molecules move.

WARMUP Rice on the Scale — predict first

Your teacher sets a rigid plastic lid, slightly tilted, on a scale and zeroes the reading. Then a steady stream of dry rice pours from a cup about half a meter above. Each grain strikes the lid for a few milliseconds and bounces off the low edge into a bin — **nothing piles up on the lid**.



The apparatus. Commit to a prediction before the pour starts.

Prediction — circle one. While the rice stream is falling, the scale's reading:

(A) stays at zero except brief blips — one grain is far too light to register **(B)** holds a roughly steady nonzero value, jiggling slightly, until the stream ends **(C)** climbs steadily the whole time the stream lasts

In one line, why did you pick your prediction?

____

QUICK Map the Demo onto a Gas

Fill each blank from the bank: *one gas molecule* · *the container wall* · *the steady gas pressure on that wall*

one rice grain ↔ ________________________ the tilted lid ↔ ________________________

the steady scale reading ↔ ________________________

FROM ONE BOUNCE TO A STEADY PUSH

ONE COLLISION AT A TIME

A molecule that hits a wall head-on at speed v and rebounds elastically has its momentum reversed, so the wall receives an impulse $J = 2mv$ per collision (Newton's third law — yesterday's "One Kick on the Wall"). One kick is far too small to notice. But a wall in room air receives roughly 10^{23} kicks on each square centimeter each second, and that torrent of tiny impulses adds up to a steady force:

$$F_{avg} = (\text{impulse per collision}) \times (\text{collisions per second}) \qquad P = \frac{F}{A}$$

Units carry the meaning: $(\mathrm{N{\cdot}s})(1/\mathrm{s}) = \mathrm{N}$, and $\mathrm{N/m^2}$ is the pressure unit (1 pascal, Pa).

WE DO **From One Atom to a Pressure Reading**

A helium atom ($m = 6.64 \times 10^{-27}$ kg) strikes a sensor pad head-on at 1250 m/s and rebounds at 1250 m/s. Take "toward the pad" as positive.

(a) The atom's momentum change: $\Delta p = m(v_f - v_i) =$ ______________ kg·m/s, so the pad receives $J =$ ______________ N·s per collision.

(b) The pad receives 3.0×10^{22} such collisions each second. Compute the average force on the pad, carrying units:

(c) The pad's area is 4.0 cm² = ______________ m². Compute the pressure on the pad: $P = F/A =$ ______________ N/m².

YOU DO **A Faster, Busier Pad**

Helium atoms strike a second sensor pad head-on at 1700 m/s each and rebound elastically, 6.0×10^{22} collisions each second, on a pad of area 8.0 cm². Find the impulse per collision, the average force on the pad, and the pressure on the pad. Carry units through every step.

Where the $\frac{3}{2}$ in $KE_{avg} = \frac{3}{2}k_B T$ comes from (promised yesterday): run today's impulse-times-rate bookkeeping over a whole box of molecules — moving in three independent directions, not all head-on — and the collision force totals to $P \cdot V = \frac{1}{3} N\, m\, v_{rms}^2$. Measured dilute gases obey $P \cdot V = N\, k_B\, T$ (the ideal gas law, formally introduced later this unit). Setting the two equal forces $\frac{1}{2} m\, v_{rms}^2 = \frac{3}{2} k_B T$: one $\frac{1}{2} k_B T$ of translational KE for each of the three directions of space.

THE TWO KNOBS THAT SET THE PRESSURE

TWO KNOBS

The chain $F = J \times \text{rate}$ exposes what pressure depends on — think of two knobs, one per factor:

Knob 1 — impulse per collision ($J = 2mv$): grows in proportion to molecular speed.

Knob 2 — collision rate: faster molecules return to the wall sooner (rate $\propto v$), and more molecules in the same volume means proportionally more arrivals (rate $\propto N$).

Heat a rigid, sealed box until the kelvin temperature doubles: KE_{avg} doubles, so v_{rms} grows by $\sqrt{2}$ — each collision hits $\sqrt{2}$ harder *and* comes $\sqrt{2}$ more often. $(\sqrt{2})(\sqrt{2}) = 2$: the pressure doubles. Add molecules at fixed temperature instead: collisions come proportionally more often at unchanged strength.

QUICK Doubling Audit

Fill each cell of the table with the multiplying factor (the container's volume stays fixed in both rows):

change made to the gas	impulse per collision ×	collisions per second ×	pressure ×
kelvin temperature doubled (N fixed)			
number of molecules N doubled (T fixed)			

RMS SPEED — THE SPEED IN THE FORMULAS

v_{rms} is the speed of a molecule whose kinetic energy equals the average. Set $\frac{1}{2} m\, v_{rms}^2 = \frac{3}{2} k_B T$ and solve:

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

Kelvin only — the formula reads temperature as a molecular-energy meter, and that proportionality starts at absolute zero. And because T sits under a square root, $v_{rms} \propto \sqrt{T}$: doubling the kelvin temperature does *not* double the speed.

WE DO Helium on a Cold Morning

The helium in a weather balloon sits at –28°C. Find the rms speed of the helium atoms ($m = 6.64 \times 10^{-27}$ kg).

Step 1 — convert: $T = -28^\circ\text{C} + 273 =$ __________ K

Step 2 — compute, carrying units: $v_{rms} = \sqrt{3k_B T/m} =$ __________ m/s

YOU DO Helium in a Hot Duct

A heat-treatment duct holds helium at 217°C. **(a)** Convert the duct's temperature to kelvin. **(b)** Compute the rms speed of the duct's helium atoms, carrying units.

(c) Compare with the We Do: the kelvin temperature doubled (245 K → 490 K), and v_{rms} grew by a factor of only __________ = $\sqrt{2}$.

PRACTICE

1 Build a Pressure from Scratch

Helium atoms ($m = 6.64 \times 10^{-27}$ kg) strike a detector plate head-on at 1200 m/s and rebound elastically at the same speed, 4.5×10^{22} collisions each second, on a plate of area 5.0 cm². Find **(a)** the impulse one atom delivers to the plate, **(b)** the average force on the plate, and **(c)** the pressure on the plate. Carry units at every step.

2 Stratosphere Speed

A research balloon's helium reaches –43°C at altitude. **(a)** Convert the helium's temperature to kelvin. **(b)** Find the rms speed of the helium atoms (carry units).

3 One Tank, Three Factors

A rigid, sealed tank of helium is heated from –38°C to 197°C. **(a)** Convert both temperatures to kelvin: __________ K → __________ K. Then give the factor by which each quantity grows: **(b)** the average translational KE per atom × __________ **(c)** v_{rms} × __________ **(d)** the tank pressure × __________.

4 Fix the Claim

A student says: *"Doubling the kelvin temperature doubles the average speed of the molecules — that's why the pressure doubles."* The pressure conclusion is right but the speed claim is not. In 2–3 sentences, state what actually happens to v_{rms} when the kelvin temperature doubles, and explain — using impulse per collision and collision rate — why the pressure still doubles.

5 Pump It Up

A rigid container at constant temperature holds 3.2×10^{22} helium atoms at a pressure of 55 kPa. A pump raises the count to 6.4×10^{22} atoms at the same temperature. **(a)** Find the new pressure. **(b)** In one line: which knob changed — impulse per collision, or collisions per second?